CONVECTIVE INSTABILITY IN ICE I: APPLICATION TO CALLISTO AND GANYMEDE. A. C. Barr, R. T. Pappalardo, Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO, 80309-0392 (amy.barr@colorado.edu).

**Introduction:** Laboratory experiments measuring ice rheology suggest that it deforms under the influence of several non-Newtonian creep mechanisms, where the viscosity depends on both strain rate and temperature [Goldsbly & Kohlstedt, 2001]. Whether or not a fluid with purely temperature-dependent viscosity convects can be determined by comparing the Rayleigh number of the system to the critical Rayleigh number \( (Ra_c, o) \), which depends on rheological, thermal, and physical parameters of the fluid layer. However, in a non-Newtonian fluid where viscosity depends on the strain rate (i.e., velocity), convection can only occur if a temperature or velocity perturbation is issued to the system to lower the viscosity and permit fluid motions. Therefore, whether convection occurs in an ice I layer depends on initial conditions, in addition to rheological, thermal, and physical properties of the layer.

We show new results for a scaling between the critical Rayleigh number and perturbation amplitude for grain boundary sliding rheology. This scaling can be used to determine the conditions required to initiate convection in the ice I shell of a generic icy satellite. We use this scaling to judge the convective instability of Ganymede and Callisto’s ice shells in the absence of tidal dissipation.

**Rheology of ice I:** Deformation maps for grain sizes \( d = 0.1 \text{ mm} - 10 \text{ cm} \) using the rheology of Goldsbly & Kohlstedt [2001] are shown in Figure 1. At the level of convective stress for a 150-km-thick ice shell within Callisto or Ganymede, \( \sigma \sim 0.1 \rho g \alpha \Delta T \), \( D \sim 0.2 \text{ MPa} \), the dominant ice flow laws are grain boundary sliding (GBS) \( (Q^* = 49 \text{ kJ/mol}, n = 1.8) \) and basal slip \( (Q^* = 60 \text{ kJ/mol}, n = 2.4) \). For GBS, the viscosity of ice depends on the grain size as \( d^{1.4} \).

**Numerical Model:** We have modified the finite-element convection model Citcom [Moresi and Gurnis, 1996; Zhong et al., 1998; Zhong et al., 2000] to implement a non-Newtonian rheology for ice. The rheology is phrased in non-dimensional terms. The Rayleigh number of the system to the critical Rayleigh number \( (Ra_c, o) \) is generally defined as:

\[
Ra = \frac{\rho g \alpha \Delta T D^{(n+2)/n}}{\kappa D^2} \exp \left( \frac{\kappa D^2}{\rho g \alpha \Delta T} \right)
\]

where \( \rho \) is the ice density, \( g \) is gravity, \( \alpha \) is coefficient of thermal expansion, \( \kappa \) is thermal diffusivity, and \( D \) is the ice shell thickness. Whether or not the system can convect regardless of initial conditions depends on the activation energy and layer thickness.

Our model has been benchmarked using a Newtonian, temperature-linearized flow law with large viscosity contrasts up to \( 10^4 \) [Blankenbach et al., 1989], and with non-Newtonian flow laws with \( n = 3 \) and viscosity contrasts of \( 10^4 - 10^7 \) [Christensen, 1985]. In the vast majority of cases in the parameter space closest to the icy satellites, convective heat flux values (Nu) and internal temperatures (average T) agree with published simulations to within 1%.

**Determination of Ra_cr:** Existing convection literature generally defines \( Ra_{cr} \) for a non-Newtonian fluid as the minimum \( Ra \) where convection can occur, regardless of initial conditions [e.g., Solomatov, 1995]. Numerically, this amounts to starting the system from a pre-existing convection pattern, and watching whether convection continues after a change in \( Ra \) or rheological parameters. This definition of \( Ra_{cr} \) addresses the question: if the system were convecting initially, would it continue to convect after changing \( Ra \) or \( Q^* \)?

This definition of \( Ra_{cr} \) does not address the question of whether we expect convection to initiate in an icy satellite, where convection may starts from a conductive equilibrium. Modest temperature perturbations could arise from physical processes such as localized tidal dissipation or aggregations of rock.

We use an initial temperature field of form:

\[ T(x, z) = T_s + \frac{1}{2} (T_m - T_s) + \frac{\delta T^*}{2} \cos(\pi x) \sin(\pi z) \]

where \( \delta T^* \) is the amplitude of initial temperature perturbation. In our model, the velocity field is initially calculated based on thermal buoyancy, so a temperature perturbation results in a velocity perturbation. We consider a range of \( \delta T^* \) from 0.005 to 0.1. Parameter sets where the amplitude of the initial perturbation grows with time are judged to be convectively unstable; if the perturbation decays with time and the system returns to conductive equilibrium, the layer and does not convect.

We define \( Ra_{cr} \) as the minimum \( Ra \) permitting convection for a given \( \delta T^* \). If there is no motion in the fluid initially, the viscosity of the layer becomes infinite, so we expect \( Ra_{cr} \to \infty \) as \( \delta T^* \to 0 \). We also expect \( Ra_{cr} \) to approach a finite value as \( \delta T^* \to 1 \), because this is roughly similar to starting the system from an initially convective temperature field. This value is denoted as \( Ra_{cr,1} \), and \( Ra < Ra_{cr,1} \) implies that the system cannot convect regardless of initial conditions.

Figure 2 shows where convection occurs for the pairs of \( Ra \) and \( \delta T^* \) used in this study. A fit to the \( Ra_{cr} \) data indicates that \( Ra_{cr} = Ra_{cr,1} (\delta T^*/\Delta T)^{\beta} \). Based on stability analyses...

1. Introduction

2. Results

3. Discussion

4. Conclusion

References

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